

# Tunneling into the edge of a compressible Quantum Hall state

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We present a composite fermion theory of tunneling into the edge of a compressible quantum Hall system. The tunnel conductance is non-ohmic, due to slow relaxation of electromagnetic and Chern-Simons field disturbances caused by the tunneling electron. Universal results are obtained in the limit of a large number of channels involved in the relaxation. The tunneling exponent is found to be a continuous function of the filling factor  $\nu$ , with a slope that is discontinuous at  $\nu = 1/2$  in the limit of vanishing bulk resistivity  $\rho_{xx}$ . When  $\nu$  corresponds to a principal fractional quantized Hall state, our results agree with the chiral Luttinger liquid theories of Wen, and Kane, Fisher and Polchinski.

*Introduction:* The edge of a Quantum Hall (QH) system plays a central role in charge transport, because the edge states carry Hall current [1]. Also, for odd-denominator Landau level filling factors  $\nu$  that correspond to incompressible quantized Hall states, the excitations on the edge form a strongly interacting one-dimensional system, which has drawn a lot of interest [2,3]. The theoretical picture of the QH edge is based on chiral Luttinger liquid models, involving either one or several chiral modes which may travel in the same or in opposite directions.

Another important part of the QH theory is the fermion-Chern-Simons approach, which can describe compressible QH states at even-denominator fractions such as  $\nu = 1/2$ , as well as the incompressible states [4,5]. In this approach, the fractional QH effect is mapped onto the integer QH problem for new quasiparticles, composite fermions [6], which interact with a statistical Chern-Simons gauge field such that each fermion carries with it an even number  $p$  of quanta of the Chern-Simons magnetic flux. The structure of the edge can then be obtained from Landau levels for composite fermions in the average residual magnetic field [7].

The physics of the edge can be probed experimentally by a tunneling conductivity measurement. Soon after first attempts to study tunneling between edges of incompressible QH states in conventional gated 2D structures [8], a new generation of 2D systems was developed by using the cleaved edge overgrowth technique [9]. In these structures it is possible to study tunneling into the edge of a 2D electron gas from a 3D doped region. It is believed that the 2D electron system has a sharp edge, and that the confining potential is very smooth, with residual roughness of an atomic scale. With the advantage of the high quality of the system, one can explore tunneling into both incompressible and compressible QH states [10]. It is found that the tunneling conductivity is non-ohmic,  $I \sim V^\alpha$ , for  $V > 2\pi k_B T/e$ , where the exponent  $\alpha$  is a continuously decreasing function of the filling factor

$\nu$  [11].

The Luttinger liquid theories predict a power law conductivity, in agreement with the experiment [10]. However, such theories can be constructed only for specific incompressible densities. Furthermore, in the construction there is no continuity in the filling factor, because the number of chiral modes, their propagation direction, and the couplings change from one incompressible state to the other. Thus, it is of interest to develop an alternative theory capable of treating both compressible and incompressible states on a similar footing.

In this paper we present a theory of tunneling into the QH edge based on the composite fermion picture. We find that under certain conditions the tunneling exponent depends only on the conductivity and interaction in the bulk, and is insensitive to the detailed structure of the edge. In this case the main effect results from the relaxation of electromagnetic disturbance caused by tunneling electron, in which we include charge and current densities as well as the Chern-Simons field. The characteristic times and spatial scales involved in the dynamics are very large, which makes it possible to express the leading behavior in terms of measurable electromagnetic response functions, the longitudinal and Hall conductivities. Tunneling exponents can then be found as a function of the filling factor.

Let us mention that tunneling density of states in a different model of a compressible edge was studied recently by Conti and Vignale [20], by Han and Thouless [21], and by Han [22]. These works focus on the case of electron density profile in which the density is smoothly decreasing from the bulk value  $\nu_{\text{bulk}}$  to zero at the boundary. It is assumed that the width of the region in which the density is changing is many magnetic lengths. For such an edge there are many hydrodynamical modes whose contribution to the tunneling density of states must be taken into account. The tunneling exponent obtained in this model is much larger than  $1/\nu_{\text{bulk}}$ .

*Main results:* In the calculation, we assume that the number of flux quanta carried by composite fermions is  $p = 2$  for  $1/3 < \nu < 1$  and  $p = 4$  for  $1/5 < \nu < 1/3$ . Also, for simplicity, we assume that the composite fermions have “bare” conductivities  $\rho_{xx}^{(0)}$  and  $\rho_{xy}^{(0)}$  which are constants that may depend, for instance, on the density, but which are independent of temperature. The measured resistivities are then  $\rho_{xy} = \rho_{xy}^{(0)} + ph/e^2$  and  $\rho_{xx} = \rho_{xx}^{(0)}$ . The theory predicts the power-law  $I \sim V^\alpha$ , with

$$\alpha = 1 + \frac{2e^2}{\pi h} \left[ \theta_H \rho_{xy} - \theta_H^{(0)} \rho_{xy}^{(0)} \right] + \frac{e^2 \rho_{xx}}{\pi h} \ln \frac{\kappa_0 \sigma_{xx}}{\kappa \sigma_{xx}^{(0)}} \quad (1)$$

where  $\theta_H = \tan^{-1} \rho_{xx}/\rho_{xy}$  is the Hall angle,  $\theta_H^{(0)}$  is the corresponding bare Hall angle,  $\kappa$  is the compressibility, and  $\kappa_0 = m^*/2\pi\hbar^2$  is the bare composite-fermion compressibility, determined by the effective mass  $m^*$ , which we treat as a constant. We have assumed, for simplicity, a short-range repulsion between the electrons (see Fig. 1).

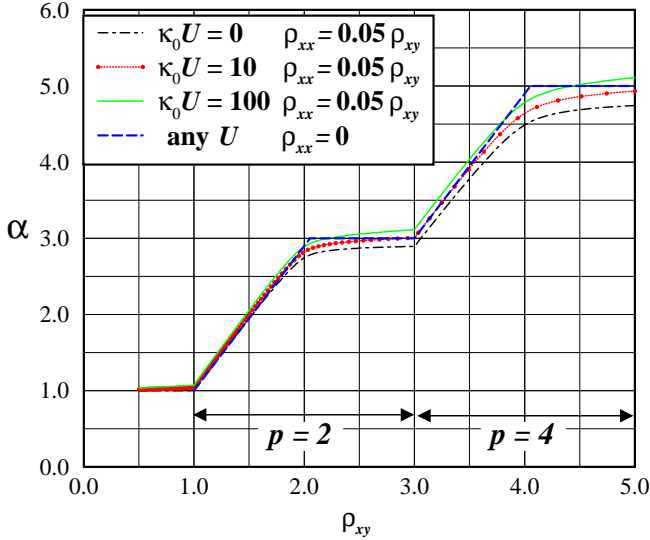


FIG. 1. The tunneling exponent (1) is shown as a function of  $\rho_{xy}$  with the composite fermion flux  $p = 2$  for  $1 \leq \rho_{xy} \leq 3$  and  $p = 4$  for  $3 \leq \rho_{xy} \leq 5$ . Constant Hall angle is assumed. For  $\rho_{xx} = 0.05\rho_{xy}$  the exponent is plotted for three values of the model short-range interaction  $U = 1/\kappa - 1/\kappa_0$ . At  $\rho_{xx} = 0$  the exponent is universal (no  $U$ -dependence), but at finite  $\rho_{xx}$  it can be either bigger or smaller than the universal result, depending on the interaction strength  $\kappa_0 U$ .

It is interesting to compare this result with theories of tunneling into the edge of an incompressible quantized Hall state, where the edge is described as a single- or multi-channel chiral Luttinger liquid. For Jain filling fractions  $\nu = n/(np+1)$  with positive integer  $n$  and even  $p$ , there are  $n$  edge modes, all traveling in the same direction. In this case, Wen [2] found universally  $\alpha = p + 1$ . By contrast, for states such as the Jain fractions with negative  $n$ , where the edges have modes going in oppo-

site directions,  $\alpha$  may depend on the form of the interaction. Nevertheless, Kane, Fisher and Polchinski [3] found in this case that if there is sufficient scattering between the channels, the system scales to a universal limit, with  $\alpha = p + 1 - 2/|n|$ .

Our result (1), taken at large Hall angle ( $\theta_H = \theta_H^{(0)} = \pi/2$ ), agrees with the chiral Luttinger liquid theories discussed above, when  $\nu$  corresponds to a Jain fraction. Specifically, with

$$\rho_{xy} = (p + 1/n) \frac{h}{e^2}, \quad \rho_{xy}^{(0)} = \frac{h}{ne^2}, \quad \rho_{xx} = \rho_{xx}^{(0)} = 0, \quad (2)$$

substituted in Eq. (1), one gets  $\alpha = 1 + |p + 1/n| - 1/|n|$ , in agreement with the universal tunneling exponents predicted by Wen and by Kane et al.

However, the theory presented below is not restricted to the incompressible states. The tunneling exponent (1) is a continuous function of the filling factor  $\nu$ . It is interesting, however, that  $\alpha$  has cusp-like singularities at  $\nu = 1/2$  and  $\nu = 1/4$ , in the limit  $\rho_{xx} = 0$ . To understand this, consider the vicinity of  $\nu = 1/2$ , where the QH state can be described as a Fermi liquid of composite fermions carrying two flux quanta each, and exposed to “residual” magnetic field  $\delta B = 2 - \nu^{-1}$ . At  $\nu < 1/2$  the residual field direction coincides with total field, and all edge modes propagate in the same direction [7]. On the other hand, at  $\nu > 1/2$ , the structure of the edge is qualitatively different, consisting of modes going in opposite directions. This effect makes  $\nu = 1/2$  a singular density. Of course, scattering by disorder will smear the singularity. However, it is interesting that the change in the tunneling exponent (1) resulting from finite  $\rho_{xx}$  can be either positive or negative, depending on the value of  $\kappa_0/\kappa$  (see Fig. 1).

*Semiclassical Green’s function:* We show below that the many-body effect on the tunneling rate arises mainly from the interaction of tunneling electron with slow electrodynamical modes. The low-energy QH physics can be described [4] by transforming electrons into composite fermions interacting via statistical Chern-Simons gauge field  $a_\mu(\mathbf{r}, t)$  ( $\mu = 0, 1, 2$ ). At low energies, the electrodynamical modes can be derived from an effective action  $S_{eff}[a_\mu]$  for  $a_\mu$  [14]. To evaluate the effect of slow fluctuations of  $a_\mu(\mathbf{r}, t)$  on the electron Green’s function, we write it as the average:

$$G_{\mathbf{r}\mathbf{r}'}(\tau) = \frac{\int Da_\mu G_{\mathbf{r}\mathbf{r}'}(\tau, a_\mu) e^{-S_{eff}[a_\mu]}}{\int Da_\mu e^{-S_{eff}[a_\mu]}} \quad (3)$$

Here  $G_{\mathbf{r}\mathbf{r}'}(\tau, a_\mu)$  is the electron Green’s function for a given configuration of the gauge field. For evaluating the tunneling current, we will only need  $G_{\mathbf{r}\mathbf{r}'}(\tau)$  for  $\mathbf{r} = \mathbf{r}'$ , which can be expressed in terms of the composite fermion Green’s function as:

$$G_{\mathbf{r}\mathbf{r}}(\tau, a_\mu) = e^{-i \int_0^\tau a_0(\mathbf{r}, t) dt} G_{\mathbf{r}\mathbf{r}}^{CF}(\tau, a_\mu). \quad (4)$$

The exponential factor represents the action associated with an extra flux  $p\Phi_0$ , which must be added at  $t = 0$  and removed at  $t = \tau$ , to account for the difference between the creation operators for an electron and a composite fermion. Note that this factor makes the electron Green's function gauge invariant, while the composite fermion Green's function itself is not gauge invariant.

We employ a semiclassical approximation for  $G_{\mathbf{r}\mathbf{r}}^{CF}(\tau, a_\mu)$ . To motivate it, think of an injected electron which rapidly binds  $p$  flux quanta and turns into a composite fermion. The latter moves in the gauge field  $a_\mu$  and picks the phase

$$\phi[a_\mu] = \int a^\mu(\mathbf{r}, t) j_\mu^{(CF)}(\mathbf{r}, t) d^2\mathbf{r} dt, \quad (5)$$

where  $j_\mu^{(CF)}(\mathbf{r}, t)$  is the current describing spreading of *free* composite fermion density. Semiclassically in  $a_\mu(\mathbf{r}, t)$  one writes

$$G_{\mathbf{r}\mathbf{r}}^{CF}(\tau, a_\mu) = e^{i\phi[a_\mu]} G^0(\tau), \quad (6)$$

where  $G^0(\tau)$  is the composite fermion Green's function in the absence of the slow gauge field. Note that fast fluctuations of  $a_\mu$  are included in  $G^0(\tau)$  through renormalization of Fermi-liquid parameters. Below we average the expression (6) over the slow gauge field (3) and find  $G_{\mathbf{r}\mathbf{r}}(\tau) = \tau^{-\alpha}$ .

Let us discuss the physical meaning of the phase factor approximation (6). It accounts correctly for the action due to charge and gauge field relaxation during tunneling. One notes the similarity to the classic infrared catastrophe in Quantum Electrodynamics, where it has been shown that the phase approximation gives correct infrared asymptotics of electron Green's function [16]. Also, similar ideas were used in the problems of tunneling coupled to an electromagnetic environment [17].

The most important correction to (6) is due to the shakeup of the composite Fermi system due to short range interactions with the injected electron. This "orthogonality catastrophe effect" [12] can be estimated by introducing scattering channels for a given field  $a_\mu(\mathbf{r}, t)$ . (For example, for the incompressible QH states the channels are edge modes.) If the gauge field is concentrated on the scale  $L$ , it excites  $N \simeq k_F L$  composite fermion channels. In terms of scattering phases  $\delta_i$ , ( $i = 1, \dots, N$ ), the orthogonality contribution is given by  $\exp(-\sum_i \delta_i^2 / \pi^2 \ln(t/t_0))$ , where  $t_0$  is a short time cutoff. To estimate  $\delta_i$  we use Friedel sum rule [12]:  $\sum_i \delta_i = \pi$  (it means that we inject unit charge). Hence,  $\delta_i \sim \pi/N$  and the orthogonality correction to the exponent  $\alpha$  is  $\simeq \text{const}/N \ll 1$ .

The effective number of channels  $N$  involved in charge relaxation is certainly large in any compressible state,

i.e., at finite  $\sigma_{xx}$ . However, the situation is more subtle in an incompressible state with small number of edge modes, where there are two possibilities. First, if all edge modes propagate in the same direction, the orthogonality catastrophe effect is simply absent, because in order to pick up the phase  $\delta_i$  a particle must travel *across* the perturbation. This is the case, for instance, at Jain fractions  $\nu = n/(2n+1)$  with  $n > 0$ , where our result is exact. On the other hand, if there is a small number of edge modes going in both directions, the orthogonality correction is significant and may lead to large deviations of  $\alpha$  from the universal value. In this way we can understand the non-universal values of  $\alpha$  obtained for Jain fractions with negative  $n$  in the absence of impurity scattering [2,3].

*Averaging over slow modes:* Putting together Eqs. (4)-(6) and Eq. (3) we get

$$G_{\mathbf{r}\mathbf{r}}(\tau) = G^0(\tau) \frac{\int Da_\mu e^{-S_{eff}[a_\mu] + i \int j_\mu a^\mu d^2x dt}}{\int Da_\mu e^{-S_{eff}[a_\mu]}} \quad (7)$$

where the current  $j_\mu = j_\mu^{(CF)} + j_\mu^{(flux)}$ , according to Eq.(4), includes the gauge field flux term

$$j_\mu^{(flux)}(\mathbf{r}, t) = -\delta_{\mu 0} \delta(\mathbf{r}) (\theta(t) - \theta(t - \tau)). \quad (8)$$

By expanding  $S_{eff}[a_\mu]$  up to the second order in  $a_\mu$  and averaging over Gaussian fluctuations in (7), we get  $G_{\mathbf{r}\mathbf{r}}(\tau) = G^0(\tau) \exp(-S(\tau))$  where

$$S = \frac{1}{2} \int d^3x d^3x' j^\mu(x) j^\nu(x') \mathcal{D}_{\mu\nu}(x, x'). \quad (9)$$

Here  $\mathcal{D}_{\mu\nu}(x, x') = \langle a_\mu(x) a_\nu(x') \rangle$  is the correlator of gauge field fluctuations ( $x = (\mathbf{r}, t)$ ). One may interpret  $S(\tau)$  as the semiclassical action of spreading charge [18]. *Calculation:* Hereafter we concentrate on the case of local conductivity tensor  $\hat{\sigma}$ , i.e., assume  $kl, \omega l/v_F < 1$ , where  $l$  is the mean free path of composite fermions. Then the current  $j_\mu^{(CF)}$  can be found from the diffusion equation with a particle source at  $\mathbf{r} = 0$  [19]

$$\partial_t j_0 + \nabla_i (D_{ij} \nabla_j j_0) = \delta^{(2)}(\mathbf{r}) (\delta(t) - \delta(t - \tau)), \quad (10)$$

Here the *noninteracting* composite fermion diffusion tensor  $\hat{D}$  is given by the Einstein relation:  $\hat{D} = \kappa_0 \hat{\sigma}^{(0)}$ . The current is determined by the diffusion law  $\mathbf{j} = -\hat{D} \nabla j_0$ .

To get the electron Green's function at the QH state edge, we choose the  $x$ -axis along the edge and assume that tunneling occurs at  $x = 0, y = 0$  (2DEG occupies the half-space  $y > 0$ ). Then Eq.(10) must be solved with the boundary condition  $j_y(y = 0) = 0$ . To treat Eq.(10), it is convenient to use Fourier transform with respect to  $x$  and  $t$ :  $j_0(k, y, \omega) = \int j_0(x, y, t) e^{ikx - i\omega t} dx dt$ . One can write a formal solution to Eq.(10):

$$j_0 = \frac{1}{i\omega + \nabla_i D_{ij} \nabla_j} J(\omega) \delta(y), \quad J(\omega) = (1 - e^{-i\omega\tau}) \quad (11)$$

Eq.(11) holds also in the general case if  $\hat{D}$  is treated as a nonlocal operator.

To find the gauge field correlator  $\mathcal{D}_{\mu\nu}$ , we choose the gauge  $a_0 = 0$  and, following [4], write

$$\hat{D} = \left( \hat{K}_0 + \hat{U}^{-1} \right)^{-1}. \quad (12)$$

Here

$$\hat{U}_{ij} = \nabla_i \frac{U(\mathbf{r} - \mathbf{r}')}{\omega^2} \nabla_j' + \frac{2\pi\hbar ip}{e^2\omega} \epsilon_{ij} \delta(\mathbf{r} - \mathbf{r}') \quad (13)$$

is the bare interaction and  $\hat{K}_0$  is current correlation function which can be found, e.g., using fluctuation-dissipation theorem:  $\hat{K}_0 = \omega^2(i\omega + (\nabla\hat{D})\nabla)^{-1}\hat{\sigma}_0$ . For simplicity, we assume short-range electron interaction  $U(\mathbf{r}) = U\delta(\mathbf{r})$ .

Let us remark that one has to be careful in substituting (12) into (9), since the flux current does not couple to the Coulomb part of the gauge field. Hence, one must subtract a flux term from the longitudinal current:  $\nabla \cdot \mathbf{j} \rightarrow \nabla \cdot \mathbf{j} - \partial_t j_0^{(flux)}$ .

After that, we decompose the product of two operators into partial fractions:

$$S(\tau) = \frac{1}{2} \left\langle J \left| (\nabla(\hat{\sigma}\nabla))^{-1} \left( \frac{1}{i\omega + (\nabla\hat{D})\nabla} - \frac{1}{i\omega + (\nabla\hat{D})\nabla + \omega^2\hat{\sigma}_0\hat{U}} \right) \right| J \right\rangle \quad (14)$$

(angular brackets denote integration over coordinates.) Now, the operators in (14) must be inverted, taking into account the boundary condition  $j_y(y=0) = 0$ . After some tedious algebra one finally arrives at

$$S(\tau) = \sum_{\omega, k} \frac{|J(\omega)|^2}{2\omega} \left( \frac{1}{\sigma_{xx}q + i\sigma_{xy}k} - \frac{1}{\sigma_{xx}^{(0)}q_0 + i\sigma_{xy}^{(0)}k} \right), \quad (15)$$

where  $\sigma_{xx}$  and  $\sigma_{xy}$  are components of the measured conductivity tensor [4]  $\hat{\sigma} = \hat{\rho}^{-1}$ , while  $q^2 = k^2 + i\kappa\omega/\sigma_{xx}$ , and  $q_0^2 = k^2 + i\kappa_0\omega/\sigma_{xx}^{(0)}$ . Integrating (15) over  $\omega$  and  $k$  one gets  $S(\tau) = (\alpha - 1)\ln(\tau/\tau_0)$  with  $\alpha$  defined by Eq. (1). Here  $\tau_0$  is a short-time cutoff.

We also assume that composite fermions are well defined quasiparticles and thus  $G_{CF}^0(\tau) \sim \tau^{-1}$ . To justify that, one may consider the self-energy  $\Sigma(\epsilon, \xi)$  of a composite fermion arising from the fluctuations of the gauge field. The self-energy is known to be a singular function of energy  $\epsilon$  but not of  $\xi = v_F(|\mathbf{p}| - p_F)$  (see [4], [15]). Because of that, the pole of the equal point Green's function  $G_{\mathbf{r}\mathbf{r}}^0(\tau)$  remains intact. This situation is similar to that for electron-phonon interaction, where  $\Sigma$  does not depend on  $\xi$  and thus does not affect tunneling current.

We remark that the point  $\mathbf{r}$  is taken to be close to the edge of the system, and the form of  $G^0$  holds in this case even for  $\delta B \neq 0$ .

Putting everything together, we have  $G(\tau) \sim \tau^{-\alpha}$ . The tunneling current is then given by [13]:

$$I(V) \sim \text{Im} \int_0^\infty G(\tau) \frac{e^{ieV\tau}}{\tau} d\tau \sim V^\alpha \quad (16)$$

where  $V$  is the voltage across the barrier.

Now we check that the number of channels is big. The gauge field disturbance caused by the tunneling electron decays in the bulk as  $\exp(-qy)$ . Hence, after time  $t$  the perturbation spreads over  $y_0(t) \simeq \sqrt{\kappa\sigma_{xx}t}$ . To estimate spreading along the edge, one compares two terms in the denominator of (15) assuming  $k \sim x_0^{-1}$ , which gives  $x_0(t) \simeq \sigma_{xy}/\sigma_{xx}y_0(t)$ . (Note that charge spreading is effectively one-dimensional if  $\sigma_{xy} \gg \sigma_{xx}$ .) To estimate the number of scattering channels parallel to the boundary, one takes  $L \sim y_0(t)$  as the spatial scale, and gets  $N \sim y_0(t)k_F$ . Since  $y_0(t)$  is large at large  $t$  (i.e., at sufficiently low bias  $V \sim \hbar/t$ ) we indeed have  $N \gg 1$ .

*Conclusion:* The above composite fermion theory of tunneling into a Quantum Hall edge treats on a similar footing both compressible and incompressible states. We have shown that tunneling  $I - V$  characteristic is non-ohmic and obeys a power law for both compressible and incompressible Quantum Hall edges. The power law exponent is expressed in terms of  $\rho_{xx}$  and  $\rho_{xy}$ , with a weak dependence on the ratio  $\kappa/\kappa_0$ , which depends on the form of the interaction. The exponent is a continuous function of  $\rho_{xy}$ , with cusp-like singularities at  $\nu = 1/2$  and  $1/4$  in the limit where  $\rho_{xx} = 0$ . The physical origin of the singularity is that the effective magnetic field seen by composite fermions changes sign at  $\nu = 1/2$  and  $1/4$ .

It is shown that the model is robust for compressible QH states when the number of scattering channels involved in charge relaxation is large. At the Jain fractions  $\nu = n/(2n \pm 1)$  the results agree with the chiral Luttinger liquid theories.

The calculated exponent is a monotonically decreasing function of filling fraction, in qualitative agreement with the recent experiment by Chang et al [10,11]. However, the theoretical value of  $\alpha$  is larger than that observed in the experiments, particularly near  $\nu = 1/2$ . The discrepancy might be explained if the density near the edge of the 2D electron system is actually somewhat larger than that in the bulk.

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